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LIFETIME EMPLOYMENT AND AN INTERNATIONAL MIXED COURNOT DUOPOLY GAME WITH A FOREIGN JOINT-STOCK FIRM

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Abstract: In economic theory, firms are often assumed to maximise profits. However, in the real world, not all firms strictly follow profit maximisation. Some firms have other objectives beyond just maximising profits. Such types of firms include state-owned welfare-maximising firms and joint-stock income-per-unit-of-capital-maximising firms. In the realm of economic theory literature, economists often delve into the behaviour of state-owned public firms, while the behaviour of joint-stock private firms receives comparatively less attention. This paper examines an international mixed duopoly game where a state-owned public firm competes against a foreign joint-stock private firm. The game unfolds in two stages. In the first stage, each firm independently and simultaneously decides whether to offer lifetime employment as a strategic commitment device. If a firm offers this strategic commitment device, it selects an output level and enters into a lifetime employment contract with the necessary number of employees to achieve the output level. In the second stage, each firm independently and simultaneously chooses its actual output. Analysis of the international mixed duopoly model reveals that there exists an equilibrium in which the state-owned firm offers lifetime employment while the foreign joint-stock firm does not. Consequently, aggressive actions by the state-owned firm against the foreign joint-stock firm can lead to domestic social welfare maximisation.

Keywords: Foreign joint-stock firm; International mixed duopoly; Lifetime employment; State-owned firm

JEL classification: C72, D21, F23, L30

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1. INTRODUCTION

In the realm of mixed market models, economists often explore the behaviour of state-owned welfare-maximising public firms. For example, Lu (2006) examines a mixed duopoly game where a welfare-maximising public firm competes against a profit-maximising private firm, using a linear-city location-then-price model with linear transportation costs and shows that there is no pure strategy subgame perfect Nash equilibrium for the game. Bárcena-Ruiz and Garzón (2003) investigate a mixed duopoly model in which a welfare-maximising public firm coexists with a profit-maximising private firm and show that both the public firm and the private firm want to merge if and only if the percentage of the shares owned by the government takes an intermediate value and the substitutability of goods is relatively low. Scrimitore (2014) investigates simultaneous and sequential competition between a welfare-maximising public firm and a profit-maximising private firm under optimal subsidies and finds that the extent of subsidy needed to attain the social optimum depends on the mode of competition (quantity or price) and whether there is public or private leadership. Xu, Lee and Wang (2018) highlight the complex interactions between privatisation, trade policies and welfare outcomes in bilateral mixed markets where a welfare-maximising public firm competes with both domestic and foreign profit-maximising private firms in each country. They find that higher social welfare can be achieved with an appropriate degree of privatisation when both governments adopt a production subsidy only and Free Trade Agreements can serve as coordination devices to address the prisoner's dilemma problem in these markets. There are many excellent further studies (see, e.g., Delbono and Denicolò, 1993; Delbono and Scarpa, 1995; Fjell and Heywood, 2002; Liu, Wang and Zeng, 2020; Ma, Wang and Zeng, 2021; Matsumura, 2003; Mujumdar and Pal, 1998; Nett, 1994; Ohnishi, 2006, 2008; Pal, 1998; Pal and White, 1998; Poyago-Theotoky, 1998; Wen and Sasaki, 2001; White, 1996; Yang and Huang, 2023; Zhang and Li, 2013).

The behaviour of joint-stock income-per-unit-of-capital-maximising firms is seldom encountered in the literature on economic theory. Meade (1972) discusses the differences in incentives, short-run adjustment and other factors between joint-stock, labour-managed and profit-maximising firms. Hey (1981) focuses on the scenario of a perfectly competitive firm that produces a single output using two inputs, labour and capital, and examines the behaviour of joint-stock, labour-managed and profit-maximising firms. Ohnishi (2010) examines a two-stage Cournot duopoly model with a profit-

maximising capitalist firm and a joint-stock firm. In the first stage, each firm non-cooperatively decides whether to provide lifetime employment as a strategic commitment. In the second stage, both firms non-cooperatively choose actual outputs. It is shown that there exist two asymmetric equilibria in which only one firm provides lifetime employment. Ohnishi (2015) investigates a three-stage mixed duopoly model where a joint-stock private firm and a state-owned public firm are allowed to offer lifetime employment as a strategic commitment. In the first stage, the state-owned firm chooses whether to provide lifetime employment or not. In the second stage, the joint-stock firm chooses whether to provide lifetime employment or not. In the third stage, each firm sets its actual quantity simultaneously and independently. Based on this analysis, introducing lifetime employment into the three-stage mixed duopoly model is beneficial for the state-owned firm. Ohnishi (2016) examines a mixed Stackelberg duopoly game in which a state-owned public firm and a joint-stock private firm can sequentially offer lifetime employment before competing in quantities. First, the joint-stock private firm decides whether to provide lifetime employment. Second, the state-owned public firm decides whether to provide lifetime employment. Third, both firms independently choose their actual outputs. It is shown that an equilibrium solution exists when only the joint-stock private firm offers lifetime employment. Ohnishi (2018) examines a mixed Cournot duopoly game in which a state-owned public firm competes against a joint-stock private firm. Each firm is allowed to offer lifetime employment as a strategic commitment device. It is shown that the equilibrium is similar to that of Ohnishi's (2015) sequential-move game. Furthermore, Ohnishi (2014) investigates the behaviour of a state-owned firm and a foreign joint-stock firm in a three-stage international mixed duopoly game. Each firm is allowed to provide a wage-rise contract as a strategic commitment. It is shown that there exists an equilibrium in which only the foreign joint-stock firm provides this strategic commitment. Ohnishi (2022) investigates an international mixed duopoly model in which a state-owned firm competes with a foreign joint-stock firm. The following situation is considered. In the first period, each firm independently and simultaneously determines its current market sales. Additionally, each firm can hold stock for the second-period market. By holding large stock, a firm can commit to large sales in the subsequent period. In the second period, each firm independently and simultaneously chooses its second-period output. Finally, at the end of the second period, each firm sells both its first-period stock and its second-period

output. It is shown that multiple stable Cournot solutions may exist in the international mixed duopoly model.

In this paper, we examine the behaviour of a state-owned public firm and a foreign joint-stock private firm in a two-stage mixed duopoly model with lifetime employment as a strategic commitment device. We consider the following situation. In the first stage, the state-owned firm and the foreign joint-stock firm independently decide whether or not to offer lifetime employment. In the second stage, each firm independently chooses its actual output. We discuss the equilibrium of the quantity-setting mixed duopoly model.

The aim of this paper is to demonstrate the impact of lifetime employment as a strategic commitment in an international mixed Cournot duopoly model where a state-owned firm competes against a foreign joint-stock firm.

The paper is structured as follows. In Section 2, we describe the formulation of the model. Section 3 provides additional explanations of the model. Section 4 discusses the equilibrium of the model. Finally, Section 5 concludes the paper.

2. MODEL

We consider a mixed duopoly market with one state-owned public firm (firm S) and one foreign joint-stock private firm (firm FJ), producing perfectly substitutable goods. For the remainder of this paper, subscripts S and FJ denote firm S and firm FJ, respectively. The market price is decided by the inverse demand function $P(Q)$, where $Q = q_S + q_{FJ}$. We assume that $P' < 0$ and $P'' < 0$.

The market is modelled using the following two-stage game. In the first stage, each firm $i (i = S,FJ)$ independently and simultaneously decides whether or not to provide lifetime employment. If firm i provides lifetime employment, then it chooses an output level $q^*_i > 0$ and enters into a lifetime employment contract with the number of employees necessary to achieve the output level. In the second stage, each firm independently and simultaneously chooses its actual output $q_i > 0$.

Therefore, domestic social welfare, which is the sum of consumers' surplus and firm S's profit, is given by

$$DW = \begin{cases} \int_0^Q P(x)dx - w(q_S) - r(q_S) - Pq_{FJ} - 2f & \text{for } q_S > q_S^*, \\ \int_0^Q P(x)dx - w(q_S^*) - r(q_S) - Pq_{FJ} - 2f & \text{for } q_S \le q_S^*, \end{cases} \tag{1}$$

where w represents the labour cost function, r is the capacity (capital) cost function, and $f > 0$ is the fixed cost. Firm S aims to maximise (1).

Furthermore, firm FJ's income per unit of capital is given by

$$V_{\mathrm{FJ}} = \begin{cases} \dfrac{P(Q)q_{\mathrm{FJ}} - w(q_{\mathrm{FJ}}) - f}{k(q_{\mathrm{FJ}})} & \text{for } q_{\mathrm{FJ}} > q_{\mathrm{FJ}}^{*}, \\ \dfrac{P(Q)q_{\mathrm{FJ}} - w(q_{\mathrm{FJ}}^{*}) - f}{k(q_{\mathrm{FJ}})} & \text{for } q_{\mathrm{FJ}} \le q_{\mathrm{FJ}}^{*}, \end{cases} \tag{2}$$

where k represents the capital input function. Firm FJ seeks to maximise (2). We assume that $k' > 0$ and $w' > 0$. Furthermore, we assume that both firms have the same technology: $w' > 0$, $w'' > 0$, $r'' > 0$, and $r'' > 0$. In the context of mixed markets, it is often assumed that both firms have the same cost functions, and marginal production costs increase (see, e.g., Bárcena-Ruiz and Garzón, 2003; Delbono and Scarpa, 1995; Fjell and Heywood, 2002; Matsumura and Kanda, 2005; Ohnishi, 2015; Pal and White, 1998; Poyago-Theotoky, 1998; White, 1996; Xu, Lee and Wang, 2018). If the **marginal cost of production remains constant or decreases,** firm S will produce an output where its price equals its marginal cost. This effectively results in **firm S supplying the entire market**, leading to a situation akin to a **public monopoly**. In this paper, we adopt **subgame perfection** as our equilibrium concept.

3. SUPPLEMENTARY EXPLANATIONS

In this section, we derive both firms' reaction functions in quantities. For $q_{\mathrm{S}} > q_{\mathrm{S}}^{*}$, firm S's reaction function is defined by

$$R_{\mathrm{S}}(q_{\mathrm{FJ}}) = \arg\max_{q_{\mathrm{S}}} \left[\int_0^{Q} P(x)dx - w(q_{\mathrm{S}}) - r(q_{\mathrm{S}}) - Pq_{\mathrm{FJ}} - 2f \right], \tag{3}$$

and for $q_{\mathrm{S}} < q_{\mathrm{S}}^{*}$, its reaction function is defined by

$$R_{\mathrm{S}}^{w}(q_{\mathrm{FJ}}) = \arg\max_{q_{\mathrm{S}}} \left[\int_0^{Q} P(x)dx - w(q_{\mathrm{S}}^{*}) - r(q_{\mathrm{S}}) - Pq_{\mathrm{FJ}} - 2f \right]. \tag{4}$$

Therefore, if firm S chooses q_{S}^{*}, then its best response is as follows:

$$R_{\mathrm{S}}^{L}(q_{\mathrm{FJ}}) = \begin{cases} R_{\mathrm{S}}(q_{\mathrm{FJ}}) & \text{for } q_{\mathrm{S}} > q_{\mathrm{S}}^{*}, \\ q_{\mathrm{S}}^{*} & \text{for } q_{\mathrm{S}} = q_{\mathrm{S}}^{*}, \\ R_{\mathrm{S}}^{w}(q_{\mathrm{FJ}}) & \text{for } q_{\mathrm{S}} < q_{\mathrm{S}}^{*}. \end{cases} \tag{5}$$

Domestic social welfare is given by (1). If firm S does not offer lifetime employment as a strategic commitment device, then its reaction function is defined by (3). On the other hand, if firm S offers lifetime employment and reduces its marginal production cost, then its reaction function is defined by

(4). Therefore, by strategic choice of this commitment device, firm S's best response becomes (5).

The equilibrium occurs when each firm maximises its objective function value with respect to its own output level, given the output level of its rival. Firm S aims to maximise domestic social welfare with respect to its own output level, given the output level of firm FJ. For $q_s > q^*_s$, the first-order and the second-order conditions are

$$P - w' - r' - P'q_{\mathrm{FJ}} = 0 \tag{6}$$

and

$$P' - w'' - r'' - P''q_{\mathrm{FJ}} < 0 . \tag{7}$$

For $q_{\mathrm{S}} < q^*_{\mathrm{S}}$, the first-order and the second-order conditions are

$$P - r' - P'q_{\mathrm{FJ}} = 0 \tag{8}$$

and

$$P' - r'' - P''q_{\mathrm{FJ}} < 0 . \tag{9}$$

Therefore, we obtain

$$R_{\mathrm{S}}{}'(q_{\mathrm{FJ}}) = \frac{P''q_{\mathrm{FJ}}}{P' - w'' - r'' - P''q_{\mathrm{FJ}}} \tag{10}$$

and

$$R_{\mathrm{S}}^{w'}(q_{\mathrm{FJ}}) = \frac{P''q_{\mathrm{FJ}}}{P' - r'' - P''q_{\mathrm{FJ}}} . \tag{11}$$

Notice that both $R_S(q_{FJ})$ and $q_{FJ} > q^*_{FJ}$ are upward sloping.

For $q_{FJ} > q^*_{FJ}$, firm FJ's reaction function is defined by

$$R_{\mathrm{FJ}}(q_{\mathrm{S}}) = \arg\max_{q_{\mathrm{FJ}}} \left[\frac{P(Q)q_{\mathrm{FJ}} - w(q_{\mathrm{FJ}}) - f}{k(q_{\mathrm{FJ}})} \right], \tag{12}$$

and for $q_{\mathrm{FJ}} < q^*_{\mathrm{FJ}}$, firm FJ's reaction function is defined by

$$R^{w}_{\mathrm{FJ}}(q_{\mathrm{S}}) = \arg\max_{q_{\mathrm{FJ}}} \left[\frac{P(Q)q_{\mathrm{FJ}} - w(q^*_{\mathrm{FJ}}) - f}{k(q_{\mathrm{FJ}})} \right]. \tag{13}$$

Therefore, if firm FJ chooses q^*_{FJ}, then its best response is as follows:

$$R^{L}_{\mathrm{FJ}}(q_{\mathrm{S}}) = \begin{cases} R_{\mathrm{FJ}}(q_{\mathrm{S}}) & \text{for } q_{\mathrm{FJ}} > q^*_{\mathrm{FJ}}, \\ q^*_{\mathrm{FJ}} & \text{for } q_{\mathrm{FJ}} = q^*_{\mathrm{FJ}}, \\ R^{w}_{\mathrm{FJ}}(q_{\mathrm{S}}) & \text{for } q_{\mathrm{FJ}} < q^*_{\mathrm{FJ}}. \end{cases} \tag{14}$$

Firm FJ seeks to maximise its objective function value with respect to its own output level, given the output level of firm S. For $q_{FJ} > q^*_{FJ}$, the first-order and the second-order conditions are

$$\left(P'q_{FJ} + P - w'\right)k - \left(Pq_{FJ} - w - f\right)k' = 0 \tag{15}$$

and

$$\left(P''q_{FJ} + 2P' - w''\right)k - \left(Pq_{FJ} - w - f\right)k'' < 0 \cdot \tag{16}$$

For $q_{FJ} < q^*_{FJ}$, the first-order and the second-order conditions are

$$\left(P'q_{FJ} + P\right)k - \left(Pq_{FJ} - w(q^*_{FJ}) - f\right)k' = 0 \tag{17}$$

and

$$\left(P''q_{FJ} + 2P'\right)k - \left(Pq_{FJ} - w(q^*_{FJ}) - f\right)k'' < 0 \tag{18}$$

Therefore, we have

$$R_{FJ}{}'(q_S) = -\frac{P''q_{FJ}k + P'\left(k - q_{FJ}k'\right)}{\left(P''q_{FJ} + 2P' - w''\right)k - \left(Pq_{FJ} - w - f\right)k''} \tag{19}$$

and

$$R^{w}_{FJ}{}'(q_S) = -\frac{P''q_{FJ}k + P'\left(k - q_{FJ}k'\right)}{\left(P''q_{FJ} + 2P'\right)k - \left(Pq_{FJ} - w(q^*_{FJ}) - f\right)k''} \tag{20}$$

Since $k'' > 0$, $k - q_{FJ}k' < 0$, so that $P''q_{FJ}k + P'\left(k - q_{FJ}k'\right)$ is positive. Also notice that both $R_{FJ}(q_S)$ and $R^{w}_{FJ}(q_S)$ are upward sloping. We find that both firms treat quantities as strategic complements.[1]

4. EQUILIBRIUM

We begin by presenting the following two lemmas.

Lemma 1: Suppose that firm i provides lifetime employment. Then the equilibrium quantity for firm i is equal to q^*_i.

We prove that if firm FJ provides lifetime employment, then in equilibrium $q_{FJ} = q^*_{FJ}$. We first consider the possibility that $q_{FJ} = q^*_{FJ}$. According to (2), when firm FJ offers lifetime employment, its objective function is denoted as

$$V_{FJ} = \frac{P(Q)q_{FJ} - w(q^*_{FJ}) - f}{k(q_{FJ})} \cdot$$

Here, since $q_{FJ} = q^*_{FJ}$, firm FJ hires additional employees. Consequently, firm FJ can enhance its objective function value by reducing q^*_{FJ}, and this

adjustment does not alter the equilibrium point when $q_{FJ} = q^*_{FJ}$. Therefore, no equilibrium arises from this situation.

Next, we consider the possibility that $q_{FJ} = q^*_{FJ}$. According to (2), firm FJ's marginal cost is represented by w'. It is impossible for firm FJ to modify its output in equilibrium because such a strategy lacks credibility. Consequently, lifetime employment does not serve as a strategic commitment device.

The proof for firm S is analogous and hence omitted.

Lemma 2: Firm *i*'s optimal output is greater when it provides lifetime employment than when it does not.

We begin by proving that firm S's welfare-maximising output is greater when it provides lifetime employment compared to when it does not. In (1), we observe that lifetime employment does not raise firm S's marginal production cost. When firm S's marginal cost is $w' + r'$, the first-order condition corresponds to (6), and when the marginal cost is r', the first-order condition becomes (8). Here, w' is positive. To satisfy (6), $P - r' - P'q_{FJ}$ must be positive. Consequently, firm *S* achieves a larger welfare-maximising output when its marginal cost is w' rather than $w' + r'$..

The proof for firm FJ is analogous and therefore omitted.

In the remainder of this section, we discuss the equilibrium of the international mixed duopoly model described in Section 2. First, we consider the scenario where only firm S can provide lifetime employment. Firm S aims to maximise domestic social welfare. Thus, if providing lifetime employment enhances domestic welfare, firm S will choose to do so; otherwise, it will refrain from offering lifetime employment.

Firm S's objective is to maximise domestic social welfare. When firm S's marginal cost is $w' + r'$, the first-order condition is expressed as (6). According to Lemma 2, firm S achieves a larger welfare-maximising output when it offers lifetime employment compared to when it does not. Suppose firm S unilaterally offers lifetime employment. Then the equilibrium occurs at a point on R_{FJ}. We consider firm S's Stackelberg leader output when each firm does not offer lifetime employment. Firm S selects q_S, and firm FJ selects q_{FJ} after observing q_S. If firm S acts as a Stackelberg leader, it maximises $DW(q_S, R_{FJ}(q_S))$ with respect to q_S. Therefore, the first-order condition for firm S's Stackelberg leader output is

$$P - w' - r' - P'q_{FJ} - P'q_{FJ}R_{FJ}' = 0 \tag{21}$$

From $P' < 0$ and $R_{\mathrm{FJ}}' > 0$, to satisfy (21), $P - w' - r' - P'q_{FJ}$ must be negative. Hence, firm S's Stackelberg leader output exceeds its Cournot output. Furthermore, $DW = \int_0^Q P(x)dx - w(q_{\mathrm{S}}) - r(q_{\mathrm{S}}) - Pq_{\mathrm{FJ}} - 2f$ is continuous and concave with respect to q_S. In the region denoted as R_{FJ}, domestic social welfare is highest at firm S's Stackelberg leader point. As a point on R_{FJ} moves farther from firm S's Stackelberg leader position, domestic social welfare decreases. Lemma 1 establishes that the equilibrium quantity is $q_S = q_S^*$.

We can now formulate the following proposition.

Proposition 1: Suppose that firm S unilaterally offers lifetime employment. Then $DW^S > DW^N$, where the superscripts '*S*' and '*N*' respectively denote the equilibrium outcome of the game where only firm S offers lifetime employment and the equilibrium outcome of the Cournot game without lifetime employment.

Second, we consider the case in which only firm FJ can provide lifetime employment. Firm FJ seeks to maximise its income per unit of capital. Consequently, it is expected that Firm FJ will adopt lifetime employment if it leads to an increase in income per unit of capital, while it will avoid this option if it results in a decrease.

Firm FJ's objective is to maximise (2). When $q_{\mathrm{FJ}} > q_{\mathrm{FJ}}^*$, the first-order condition is given by (15). According to Lemma 2, the output that maximises firm FJ's objective function value is higher when it offers lifetime employment compared to when it does not. However, if firm FJ unilaterally provides lifetime employment, the equilibrium occurs at a point on R_S. We consider firm FJ's Stackelberg leader output. Firm FJ selects q_{FJ}, and firm S selects q_S after observing q_{FJ}. If firm FJ acts as a Stackelberg leader, then it maximises $V_{FJ}(q_{FJ}, R_S(q_{FJ}))$ with respect to q_{FJ}. Therefore, the Stackelberg leader output for firm FJ satisfies the first-order condition:

$$\left(P'q_{\mathrm{FJ}} + P - w'\right)k - \left(Pq_{\mathrm{FJ}} - w - f\right)k' + P'q_{\mathrm{FJ}}R_{\mathrm{S}}' = 0 \cdot \tag{22}$$

From $P' < 0$ and $R_S' > 0$, to satisfy (22), $\left(P'q_{\mathrm{FJ}} + P - w'\right)k - \left(Pq_{\mathrm{FJ}} - w - f\right)k'$ must be positive. Consequently, firm FJ's Stackelberg leader output is lower than its Cournot output. Additionally, $V_{\mathrm{FJ}} = \left(P(Q)q_{\mathrm{FJ}} - w(q_{\mathrm{FJ}}) - f\right)/k(q_{\mathrm{FJ}})$ is continuous and concave with respect to q_{FJ}. In R_S, firm FJ's objective function value is highest at the Stackelberg leader point. As a point on R_S moves farther from firm S's Stackelberg leader position, firm FJ's objective function value decreases. Therefore, if firm FJ unilaterally provides lifetime employment, its

objective function value will be lower than in the Cournot game equilibrium without lifetime employment.

Suppose that each firm chooses q_i^*. Lemma 1 states that in equilibrium $q_i = q_i^*$. Based on (5) and (14), each firm's reaction functions have a flat segment at q_i^*. $V_{\mathrm{FJ}} = \left(P(Q)q_{\mathrm{FJ}} - w(q_{\mathrm{FJ}}) - f\right)/k(q_{\mathrm{FJ}})$ is continuous and concave with respect to q_{FJ}. Hence, firm FJ can increase its objective function value by reducing both q_{FJ}^* and q_{FJ}^*. The optimal strategy for Firm FJ involves minimising q_{FJ} and q_{FJ}^* to reach a point on R_{FJ}, given q_S and q_S^*. The equilibrium we employ is subgame perfection, with all information in the model being common knowledge. Consequently, firm S does not adjust q_S and q_S^* based on firm FJ's offer of lifetime employment. From (12), we see that firm FJ's offer of lifetime employment does not serve as a strategic commitment in R_{FJ}.

We can state the following proposition.

Proposition 2: Suppose that firm FJ provides lifetime employment, given firm S's strategy. Then $V_{\mathrm{FJ}}^S > V_{\mathrm{FJ}}^B$ and $V_{\mathrm{FJ}}^S > V_{\mathrm{FJ}}^B$, where the superscripts '*FJ*' and '*B*' respectively denote the equilibrium outcome of the game where only firm FJ offers lifetime employment and the equilibrium outcome of the game where both firms offer lifetime employment.

Third, we consider the case in which both firms have the option to offer lifetime employment. If domestic social welfare and firm FJ's income per unit of capital are lower when both firms offer lifetime employment compared to when only one firm unilaterally offers it, then there is no equilibrium where both firms choose lifetime employment. However, if this condition is not met, an equilibrium with both firms offering lifetime employment may exist.

In the first stage, each firm decides whether or not to offer lifetime employment. If firm *i* chooses to do so, it selects its output level. In the second stage, both firms independently choose their actual outputs, and both objective function values are determined.

Proposition 1 indicates that the best strategy for firm S is to offer lifetime employment when firm FJ does not. Furthermore, Proposition 2 means that the optimal choice for firm FJ is to refrain from offering lifetime employment, regardless of whether firm S does so.

The result of this study can be stated in the following proposition.

Proposition 3: In the international mixed duopoly model, there exists an equilibrium where firm S provides lifetime employment while firm FJ does not.

5. CONCLUSION

Our analysis of an international mixed duopoly game involving a state-owned public firm competing against a foreign joint-stock private firm reveals that aggressive action by the state-owned firm against the foreign joint-stock firm may lead to the maximisation of domestic social welfare. In this paper, we have explored a two-stage game. Moving forward, we plan to investigate different long-run mixed market models involving joint-stock firms.

NOTE

1. The concept of strategic complements was introduced by Bulow, Geanakoplos and Klemperer (1985).

REFERENCES

Bárcena-Ruiz, J. C., Garzón, M. B., 2003. Mixed duopoly, merger and multiproduct firms, *Journal of Economics* 80, 27-42.

Bulow, J., Geanakoplos, J., Klemperer, P., 1985. Multimarket oligopoly: strategic substitutes and complements, *Journal of Political Economy* 93, 488-511.

Delbono, F., Denicolò, V., 1993. Regulating innovative activity: the role of public firm, *International Journal of Industrial Organization* 11, 35-48.

Delbono, F., Scarpa, C., 1995. Upward-sloping reaction functions under quantity competition in mixed oligopolies, *Bulletin of Economic Research* 47, 341-346.

Fjell, K., Heywood, J. S., 2002. Public Stackelberg leadership in a mixed oligopoly with foreign firms, Australian Economic Papers 41, 267-281.

Hey, J. D., 1981. A unified theory of the behaviour of profit-maximising, labour-managed and joint-stock firms operating under uncertainty. *Economic Journal* 91, 364-374.

Liu, L. H., Wang, X. H., Zeng, C. H., 2020. Endogenous horizontal product differentiation in a mixed duopoly. *Review of Industrial Organization* 56, 435–462.

Lu, Y., 2006. Hotelling's location model in mixed duopoly. Economics Bulletin 8, 1–10.

Ma, H. K., Wang, X. H., Zeng, C. H., 2021. Location choice and costly product differentiation in a mixed duopoly. Annals of Regional Science 66, 137–159.

Matsumura, T., 2003. Stackelberg mixed duopoly with a foreign competitor, *Bulletin of Economic Research* 55, 275-287.

Matsumura, T., Kanda, O., 2005. Mixed oligopoly at free entry markets, *Journal of Economics* 84, 27-48.

Meade, J. E., 1972. The theory of labour-managed firms and of profit sharing. *Economic Journal* 82, 402-428.

Mujumdar, S., Pal, D., 1998. Effects of indirect taxation in a mixed oligopoly, *Economics Letters* 58, 199-204.

Nett, L., 1994. Why private firms are more innovative than public firms, *European Journal of Political Economy* 10, 639-653.

Ohnishi, K., 2006. A mixed duopoly with a lifetime employment contract as a strategic commitment, *Finanz Archiv* 62, 108-123.

Ohnishi, K., 2008. Strategic commitment and international mixed competition with domestic state-owned and foreign labor-managed firms. *Finanz Archiv* 64, 458-472.

Ohnishi, K., 2010. Lifetime employment contract and quantity competition with profit-maximizing and joint-stock firms. *Journal of Institutional and Theoretical Economics* 166, 462-478.

Ohnishi, K., 2014. Sequential mixed competition with a foreign joint-stock firm. *International Journal of Social Sciences and Management Studies* 1, 38-52.

Ohnishi, K., 2015. Lifetime employment and a three-stage model with state-owned and joint-stock firms. In Ohnishi, K. (Ed.), Firms' Strategic Decisions: Theoretical and Empirical Findings, Volume 1 (pp. 155-169). Sharjah, UAE: Bentham Science Publishers.

Ohnishi, K., 2016. Lifetime employment and a sequential choice in a mixed duopoly market with a joint-stock firm. *International Journal of Management, Accounting and Economics* 3, 283-295.

Ohnishi, K., 2018. Lifetime employment and mixed Cournot duopoly with state-owned and joint-stock firms. *International Journal of Management, Accounting and Economics* 5, 1-9.

Ohnishi, K., 2022. Inventory holding and a mixed duopoly with a foreign joint-stock firm. *Asian Journal of Economics and Business* 3, 1-14.

Pal, D., 1998. Endogenous timing in a mixed oligopoly, *Economics Letters* 61, 181-185.

Pal, D., White, M. D., 1998. Mixed oligopoly, privatization, and strategic trade policy, *Southern Economic Journal* 65, 264-281.

Poyago-Theotoky, J., 1998. R&D competition in a mixed duopoly under uncertainty and easy imitation, *Journal of Comparative Economics* 26, 415-428.

Scrimitore, M., 2014. Quantity competition vs. price competition under optimal subsidy in a mixed duopoly. *Economic Modelling* 42, 166-176.

Wen, M., Sasaki, D., 2001. Would excess capacity in public firms be socially optimal? *Economic Record* 77, 283-290.

White, M. D., 1996. Mixed oligopoly, privatization and subsidization, *Economics Letters* 53, 189-195.

Xu, L., Lee, S. H., Wang, L. F. S., 2018, Strategic trade and privatization policies in bilateral mixed markets. In Ohnishi, K. (Ed.), Firms' Strategic Decisions: Theoretical and Empirical Findings, Volume 3 (pp. 112-139). Sharjah, UAE: Bentham Science Publishers.

Yang, L., Huang, Z., 2023. Quality-improving licensing of an outside innovator in a mixed Cournot duopoly. *Economic Modelling* 126, 106432.

Zhang, J., Li, C., 2013. Endogenous timing in a mixed oligopoly under demand uncertainty. *Journal of Economics* 108, 273–289.